Total Marks - 120
Attempt Questions 1 – 10
All questions are equal value.
Answer each question in a SEPARATE writing booklet.

Question 1 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate, correct to three significant figures, **2**

$$\frac{2-0.35}{\sqrt{23^2+17^2}}$$

(b) Solve $x^2+2x-15=0$. **2**

(c) Express $0.\dot{1}\dot{7}$ as a fraction. **2**

(d) Simplify $\frac{1}{x^2-1}+\frac{x}{x+1}$. **2**

(e) Solve $|2x-1|>3$. **2**

(f) Find the exact value of $\tan\left(\frac{2\pi}{3}\right)$. **2**

Question 2 (12 Marks) Use a SEPARATE writing booklet.

(a) Evaluate $\lim_{x\to 0}\frac{\sin 5x}{x}$. **1**

(b) The digits 1, 2, 3 are selected at random without replacement to form a two digit number.

(i) Draw a tree diagram to illustrate all possible outcomes and list the outcomes. **2**

(ii) What is the probability that the two digit number formed is a multiple of 3? **1**

(c) A particle travels such that its displacement from O, is given by $x=t^2-6t+8$, x in metres and t in seconds.

(i) Find the particles initial position and velocity. **3**

(ii) At what time does the particle change direction? **1**

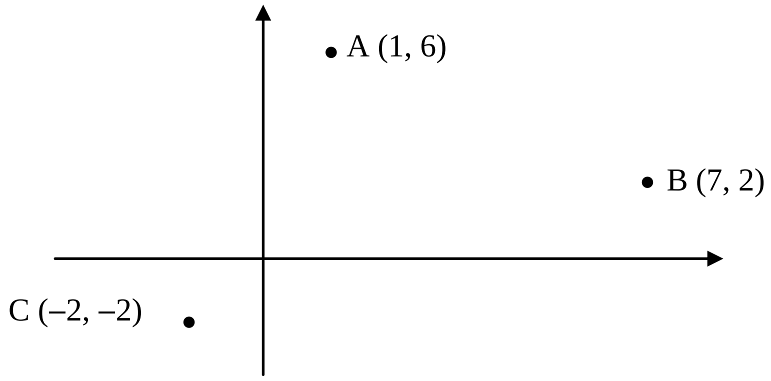
(iii) What is the total distance traveled by the particle in the first 4 seconds. **2**

Question 2 continued. **Marks**

(d) Solve for *x*: $2\sin x+\sqrt{3}=0$ for $0\le x\le 2\pi$. **2**

Question 3 (12 Marks) Use a SEPARATE writing booklet.

(a)



The diagram shows the points A (1,6), B(7,2) and C (–2,–2).

(i) Show that the gradient of AB is $-\frac{2}{3}$. **1**

(ii) Find the angle of inclination the line passing through the points AB make the x axis. (Nearest minute.) **2**

(iii) Show that equation of the line passing through C, parallel to AB is $2x+3y+10=0$. **2**

(iv) Find the perpendicular distance of the point B from the line $2x+3y+10=0$. **2**

(v) Show that the mid point of BC lies on the *x* axis. **2**

(b) A triangle has three side of lengths 6 cm, 9cm and 11 cm.

(i) Find the size of the smallest angle. **2**

(ii) Find the area of the triangle **1**

Question 4 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Evaluate $\int_0^1 \frac{3}{x+1}dx$. **2**

(b) Differentiate with respect to x:

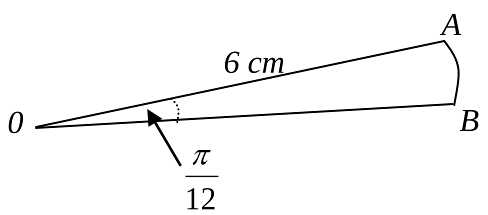
(i) $x\tan x$ **2**

(ii) $\frac{\ln x}{x}$ **2**

(c) Find the equation of the tangent to $y = \log_e x$ at the point $(e,1)$. **2**

(d) Evaluate $\sum_{n=5}^{10}(2n-3)$. **2**

(e) In the diagram, AB is an arc of a circle with centre O. The radius OA is 6 cm. The $\angle AOB$ is $\frac{\pi}{12}$. Find the exact area of the sector AOB. **2**

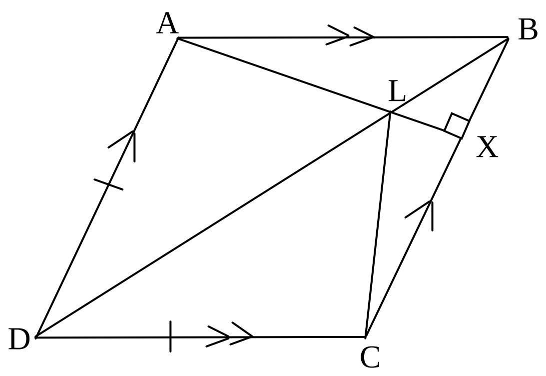


NOT TO SCALE

Question 5 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Fred has a monthly salary of \$3000. He decides to start a savings plan in which at the end of the first month he saves 5% of his salary and each successive month increases this amount by \$10. How long will it take for his savings to exceed \$6000. **4**

(b)



(figure not to scale)

ABCD is a rhombus, AX is perpendicular to BC and intersects BD at L.

(i) Copy the diagram and state why $\angle ADB = \angle CDB$. **1**

(ii) Prove that the triangles ALD and CLD are congruent. **2**

(iii) Show that $\angle DAL$ is a right angle. **1**

(iv) Hence or otherwise find the size of $\angle LCD$. **1**

(c) A ball is dropped from a high of 2 metres onto a concrete floor and rebounds to $\frac{2}{3}$ of the previous height. It continues to rebound to $\frac{2}{3}$ of the previous height for each of the following bounces.

(i) What is the maximum height reached by the ball after the third bounce? **1**

(ii) What is the maximum total distance travelled by the ball from the time it was dropped until it eventually comes to rest on the floor? **2**

Question 6 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Consider the curve given by $y = x^3 - 3x^2 - 9x + 1$.

(i) Find the coordinates of any stationary points. **4**

(ii) Determine the nature of the stationary points. **2**

(b) Draw a sketch for a function, which has the following features and indicate the nature of stationary points: **2**

$$f'(x) = 0 \text{ at } x = 0, x = 2, x = 4$$
$$f''(x) < 0 \text{ for } x < 1$$
$$f''(x) > 0 \text{ for } 1 < x < 3$$
$$f''(x) < 0 \text{ for } 3 < x < 4$$
$$f''(x) > 0 \text{ for } x > 4$$

(c) A bag contains 3 red, 4 blue and 5 yellow balls. Three balls are drawn out, one at a time without replacement.

Find the probability that:

(i) Two yellow and one red ball are drawn out in any order. **2**

(ii) At least one red ball is drawn out. **2**

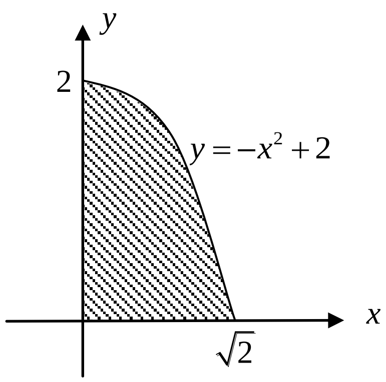
Question 7 (12 Marks) Use a SEPARATE writing booklet.

(a) For the function $y = \sin 2x$:

(i) Sketch the function for the domain $(0 \le x \le \pi)$. **2**

(ii) Find the area bounded by $y = \sin 2x$ and the x axis between $0 \le x \le \pi$. **3**

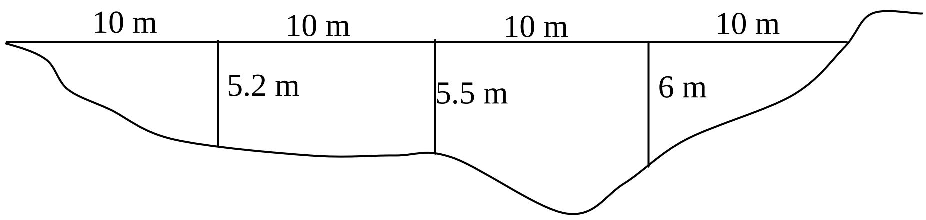
(b)



The area in the first quadrant above is rotated about the y axis. Calculate the volume of the solid formed. **3**

Question 7 continued **Marks**

(c) The diagram shows the cross-section of a river, with the depths of the river shown in metres, at 10 metre intervals. The river is 40 metres in width.



(i) Use the trapezoidal rule to find the approximate value for the area of the cross-section. **2**

(ii) Give a way of improving the accuracy for measuring the cross section and explain how this improves the accuracy. **2**

Question 8 (12 Marks) Use a SEPARATE writing booklet.

(a) An elderly marathon runner can run, such that his speed is given by

$v = 15(1 - \sin 0.15t)$ km/h up to the time he cannot run any further.

If a race starts is taken as x = 0 km and t = 0 hours:

(i) Find to nearest minute the time taken for the runners speed to drop to 5 km/h. **2**

(ii) How far to nearest metre would it taken the runner in three hours? **3**

(b) Solve the equation $3^{2x} + 2 \times 3^x - 15 = 0$. **3**

(c) Consider the equation $x^2 + (k-2)x + 4 = 0$. For what values of k does the equation have:

(i) equal roots. **2**

(ii) real and distinct roots. **2**

Question 9 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a) Solve $\frac{1}{2}\log_e(12-x)=\log_e x$ **2**

(b) A retiring couple estimate that they will need an income of \$3 750 per month to be paid at the end of each month for twenty years to see them through they retirement years. Estimating an average interest rate of 6% p.a. compounding monthly for the twenty year at which time their investment has been reduced to zero. Let P be the amount of money they invest to achieve they desired income:

(i) Show that after two months the amount of money remaining in the investment is $A_2 = P(1.005)^2 - 3750(1.005) - 3750$. **1**

(ii) Show that after n months, $A_n = P(1.005)^n - 750000\left((1.005)^n - 1\right)$. **2**

(iii) Calculate the value of P. **2**

(c) The amount A grams of a given carbon isotope in the wood of a Dark Age coffin is given by $A = A_0 e^{-kt}$ where A_0 and k are positive constants, and where t is measured in years from the time the wood was cut from a tree.

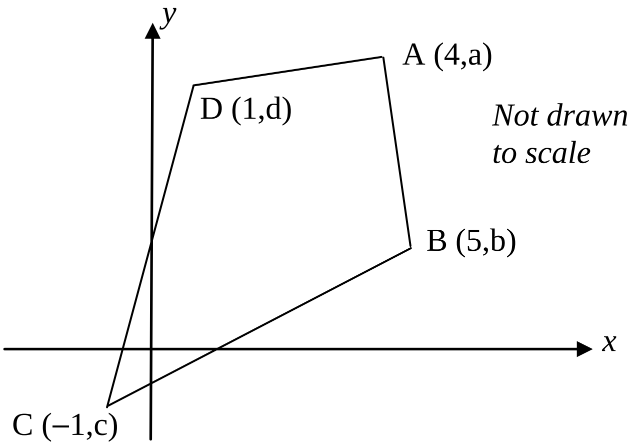
(i) Show A satisfies the equation $\frac{dA}{dt} = -kA$. **1**

(ii) Find the value of k if the amount of isotope halved every 500 years. **2**

(iii) When tested the wood only had 15% of the original amount in the living tree. How long ago was the wood cut from a tree. Give your answer to the nearest 100 years. **2**

Question 10 (12 Marks) Use a SEPARATE writing booklet. **Marks**

(a)



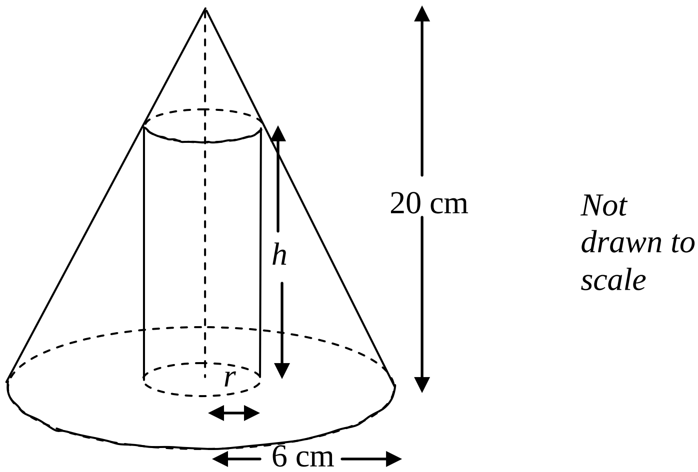
The diagram above is of a kite ABCD.

(i) Show that the equation of AC is $y = \frac{a-c}{5}(x-4)+a$. **2**

(ii) Find the mid point of BD. **1**

(iii) Hence or otherwise show that $b = \frac{2c+8a-5d}{5}$. **2**

(b)



A cylinder of radius r cm and height h cm is inscribed in a cone with base radius 6 cm and height 20 cm as in the diagram. **3**

(i) Using similar triangles, show that the volume V of the cylinder is given by:

$$V = \frac{10\pi r^2(6-r)}{3}.$$

(ii) Hence find the values of r and h for the cylinder which has maximum volume. **3**

(iii) What is the maximum volume? **1**

End of Paper.

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Year 12 Mathematics Trial 2006

Q1

(a) $= 0.057690897\ldots$
$= 0.0577$ (3 sig. fig.s)

1 Mark – correct answer
1 Mark – 3 sig. fig.s

(b) $x^2 + 2x - 15 = 0$
$\therefore (x+5)(x-3) = 0$
$\therefore x = -5$ or 3

1 Mark – correctly factorise
1 Mark – solution

(c) let $x = 0.171717\ldots$
$\therefore 100x = 17.171717\ldots$
$\therefore 99x = 17$
$\therefore x = 17/99$

OR $0.\dot{1}\dot{7} = \dfrac{\frac{17}{100}}{1 - \frac{1}{100}}$
$= 17/99$

Marking scale:
1 Mark: attempted to use correct method (but faile
2 Marks: correct solution

(d) $\dfrac{1}{x^2-1} + \dfrac{x}{x+1}$
$= \dfrac{1}{(x-1)(x+1)} + \dfrac{x(x-1)}{(x-1)(x+1)}$
$= \dfrac{1 + x^2 - x}{(x-1)(x+1)}$

1 Mark – common denominato
1 Mark – solution

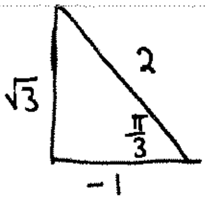
(e) $|2x-1| > 3$

$\therefore (2x-1)^2 > 9$
$\therefore 4x^2 - 4x + 1 > 9$
$\therefore 4x^2 - 4x - 8 > 0$
$\therefore 4(x^2 - x - 2) > 0$
$\therefore 4(x-2)(x+1) > 0$

+ – +
-1 2

$\therefore x < -1$ or $x > 2$

2 Marks – one per solution
Note: –1 mark if inequality was combined

(f) 

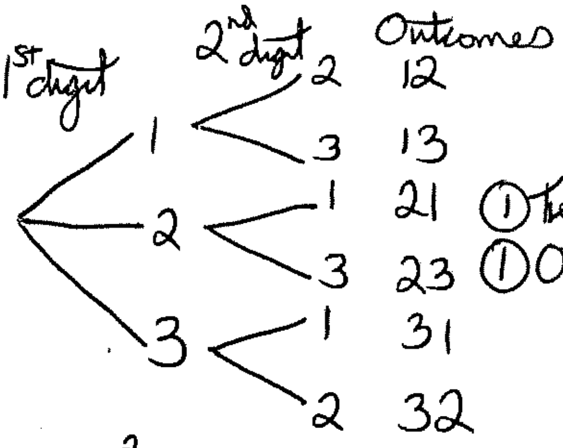
$\tan\frac{2\pi}{3} = -\sqrt{3}$

1 Mark: negative
1 Mark: $\sqrt{3}$

Yr 12 TRIAL HSC 2 UNIT 2006

(2) a) $\lim_{x\to 0}\left(\frac{\sin 5x}{x}\right) = \lim_{x\to 0}\left(5\times\frac{\sin 5x}{5x}\right) = 5\lim_{x\to 0}\left(\frac{\sin 5x}{5x}\right) = \underline{5}$ (1)

b) (i)



(1) Tree Diagram
(1) Outcomes

(ii) 12 + 21 divisible by 3

Hence $P(\div 3) = \frac{2}{6} = \underline{\frac{1}{3}}$ (1)

c) $x = t^2 - 6t + 8$

(i) $\frac{dx}{dt} = 2t - 6$. (1) $\underline{x(0) = 8m}$ (1), $\underline{\left(\frac{dx}{dt}\right)_{t=0} = -6\,m/s}$ (1)

(ii) Changes direction $\frac{dx}{dt} = 0 \quad \therefore 2t - 6 = 0 \quad \underline{t = 3s}$ (1)

(iii) $t = 0,\ x = 8$
$t = 3,\ x = 9 - 18 + 8 = -1$ (1) Splitting time interval appropriately
$t = 4,\ x = 16 - 24 + 8 = 0$

Total distance travelled $= 9 + 1 = \underline{10m}$ (1)

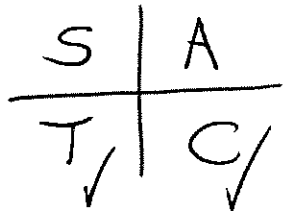
(Subtract 1 mark for all missing units)

d) $2\sin x + \sqrt{3} = 0 \qquad 0 \leq x \leq 2\pi$

$2\sin x = -\sqrt{3}$

$\sin x = -\sqrt{3}/2$

$\frac{\pi}{3}$ but 3rd + 4th quadrants (1)



$\underline{x = \frac{4\pi}{3}, \frac{5\pi}{3}}$ (1)

MARKERS COMMENTS

a) This is a standard HSC question year in year out. Far too many students made incorrect statements $\sin 5x \neq 5\sin x$ etc. even though these also give the answer 5 albeit spuriously.

b) Students were explicitly told to list outcomes, many didn't. Lots failed to read question and used replacement!

b(iii) correct probability eg from marked diagram marked here

c)(i) Lots found velocity function but not initial velocity
Many dropped negative sign failing to realise velocity is a vector quantity.

(iii) Change in direction confused many when calculating distance. Some simply added displacements at second intervals

Failure to include any units on the four answers cost students one mark

d). Most completed well.
Too many gave answers in degrees not radians

③ (i) $m_{AB} = \frac{2-6}{7-1} = \frac{-4}{6} = -\frac{2}{3}$ $\left(\text{OR } \frac{6-2}{1-7} = \frac{4}{-6} = -\frac{2}{3}\right)$ ① "SHOW" must include subtraction

(ii) $m_{AB} = \tan\alpha$ $\quad -\frac{2}{3} = \tan\alpha$ ① $\quad \alpha = \tan^{-1}\left(-\frac{2}{3}\right) = 180 - 33^\circ 41'$

$= 146^\circ 19'$ (nearest min.) ①

(iii) $m = -\frac{2}{3}$ through $(-2,-2)$

$(y - {}^{-}2) = -\frac{2}{3}(x - {}^{-}2)$ ① "SHOW" - 1 mark substitution, 1 mark rearranging

$y + 2 = -\frac{2}{3}(x+2)$

$3y + 6 = -2x - 4$ ①

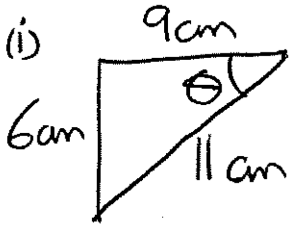
$2x + 3y + 10 = 0$

(iv) $p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}} = \frac{|2\times7 + 3\times2 + 10|}{\sqrt{2^2+3^2}}$ ① $= \frac{|30|}{\sqrt{13}} = \frac{30\sqrt{13}}{13}$ ①

(v) $M = \left(\frac{7+{}^{-}2}{2}, \frac{2+{}^{-}2}{2}\right) = \left(\frac{5}{2}, 0\right)$ ① $\therefore y = 0 \therefore$ lies on x axis.

① for statement in conclusion

b) (i)



$\cos\theta = \frac{11^2 + 9^2 - 6^2}{2\times9\times11}$ ① $= 0.8383\ldots$

$\theta = 33^\circ 02'$ (nearest min.) ①

(ii) $A = \frac{1}{2}bc\sin\theta = \frac{1}{2}\times9\times11\times\sin 33^\circ 02'$

$= 26.98\ldots$

$= 27\text{cm}^2$ (nearest whole) ①

MARKERS COMMENTS

a) (ii) Most students fail to recognise that angle of inclination is from positive x-axis ie obtuse in this case. Many didn't know m=

(iv) Lots of students don't know perpendicular distance formula

(v) Conclusion from midpoint calculation were vague in many case

b) (i) Students should recognise smallest angle is opposite smalle side, many performed calculation twice or three times.

(ii) Some used $\cos\theta$ not $\sin\theta$ in formula. Lots did not include units.

Question 4

a) $\left[3\ln(x+1)\right]_0^1$ ✓

$= 3\ln 2 - 3\ln 1$

$= 3\ln 2$ ✓

b) i) $\tan x + x\sec^2 x$ ✓ ✓

ii) $\dfrac{x \times \frac{1}{x} - 1 \times \ln x}{x^2}$

$= \dfrac{1 - \ln x}{x^2}$

c) $\dfrac{dy}{dx} = \dfrac{1}{x}$, $m = \dfrac{1}{e}$ ✓

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{e}(x - e)$

$ey - e = x - e$

$0 = x - ey$ ✓

d) $a = 7$ $d = 2$ $n = 6$ ✓

$S_6 = \frac{6}{2}(2\times 7 + 5\times 2)$

$= 3(14 + 10)$

$= 72$ ✓

e) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times 6^2 \times \frac{\pi}{12}$ ✓

$= \dfrac{18\pi}{12}$

$= \dfrac{3\pi}{2}$ ✓

(a) $150 + 160 + 170 + \cdots$

AP $\left.\begin{array}{l} a = 150 \\ d = 10 \end{array}\right]$ ✓ 1

$n = ?$

$S_n = 6000$

1 mark - establishing AP $a = 150$ $d = 10$

$$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$$

$$6000 < \frac{n}{2}\left(300 + (n-1)10\right) \checkmark \ 1$$

1 mark (sub into formula correctly) Must show inequality.

$$12000 < n(300 + 10n - 10)$$

$$12000 < 290n + 10n^2$$

$$10n^2 + 290n - 12000 > 0$$

$$n^2 + 29n - 1200 > 0$$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-29 \pm \sqrt{29^2 + (4 \times 1 \times 1200)}}{2} \checkmark$$

1 mark (sub into quad. formula correctly from their inequality) OR some other correct appropriate method

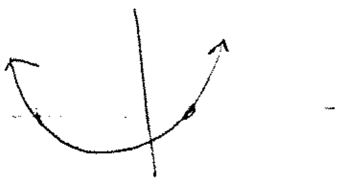
$$n = \frac{-29 \pm \sqrt{5641}}{2}$$

$$n = \frac{-104.1065909}{2} = -52.0532\ldots$$

OR

$$n = \frac{46.1065909}{2} = 23.0532\ldots$$

n is a positive number.



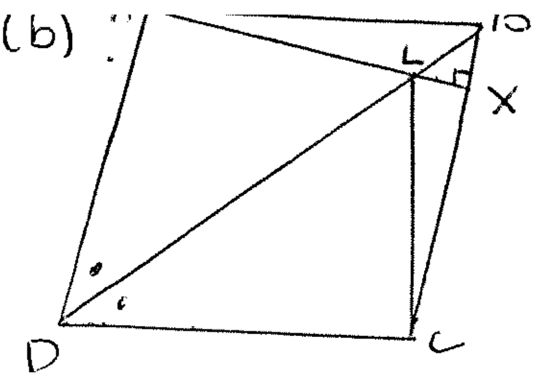
$n > 23.0532\ldots$

$\therefore n = 24$ months ✓

1 mark answer (from correct wor

An alternative method where boys showed the value of each ~~method~~ month up to 24 months was accepted. If fully correct.

(b)



(i) $\angle ADB = \angle CDB$

Diagonals bisect the vertices in a rhombus.

1 mark reason be correct.

(ii) In $\triangle ALD$ and $\triangle CLD$

$AD = CD$ (equal sides in a rhombus)

DL is common side

$\angle ADB = \angle CDB$ (diagonals bisect vertices in a rhombus (OR from (i)))

1 mark – all 3 reasons must be correct.

$\therefore \triangle ALD \equiv \triangle CLD$ (S.A.S). → 1 mark correct answer only on these lines

(iii) $\angle DAL = \angle BXA$ (alternate angles in parallel lines $AD \parallel BC$ in rhombus)

$\angle BXA = 90^\circ$ ($AX \perp$ to BC)

$\therefore \angle DAL = 90^\circ$

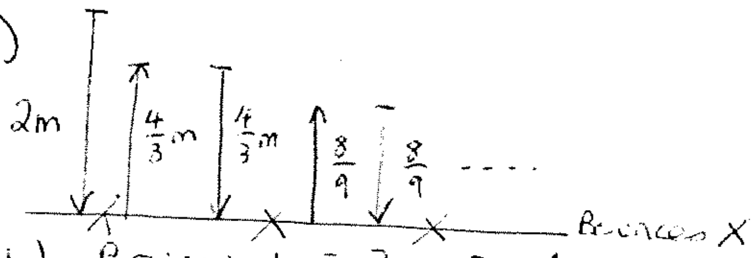
1 mark: full reason must be given. $AD \parallel BC$ must be there.

(iv) $\angle DAL = \angle DCL$ (corresponding angles in congruent triangles from part (ii))

$\therefore \angle DAL = \angle LCD = 90^\circ$

1 mark: correct reason must be given

(c)



(i) Bounce 1 $= \frac{2}{3} \times 2 = \frac{4}{3}$m

Bounce 2 $= \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$m

Bounce 3 $= \frac{2}{3} \times \frac{8}{9} = \frac{16}{27}$m

$\therefore$ height after 3rd bounce is $\frac{16}{27}$m

1 mark correct answer only.

(ii) $2 + 2\left(\frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \dots\right)$

limiting sum $a = \frac{4}{3}$ $r = \frac{2}{3}$

1 mark substitute correctly into formula.

$= 2 + 2\left(\frac{a}{1-r}\right)$

$= 2 + 2\left(\frac{4/3}{1-2/3}\right)$

$= 2 + 2(4)$

$= 10$m

$\therefore$ total distance travelled is 10m.

1 mark correct answer

Solutions Q6 Maths.

6 (a) $y = x^3 - 3x^2 - 9x$

(i) $\frac{dy}{dx} = 3x^2 - 6x - 9$

If $\frac{dy}{dx} = 0$ then $3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$\therefore x = 3$ or -1.

[4 mks correct ans.
3 mks correct x values
2 mks correct technique
1 mk correct differentiation

If $x = 3$, $y = -26$ and if $x = -1$, $y = 6$

ie. $(3, -26)$ and $(-1, 6)$ are st. pts.

(ii) $\frac{d^2y}{dx^2} = 6x - 6$ If $x = 3$ then $\frac{d^2y}{dx^2} = 12 > 0$ $\therefore$ min t.pt

If $x = -1$ then $\frac{d^2y}{dx^2} = -12 < 0$ $\therefore$ max t.pt

[1 mk each

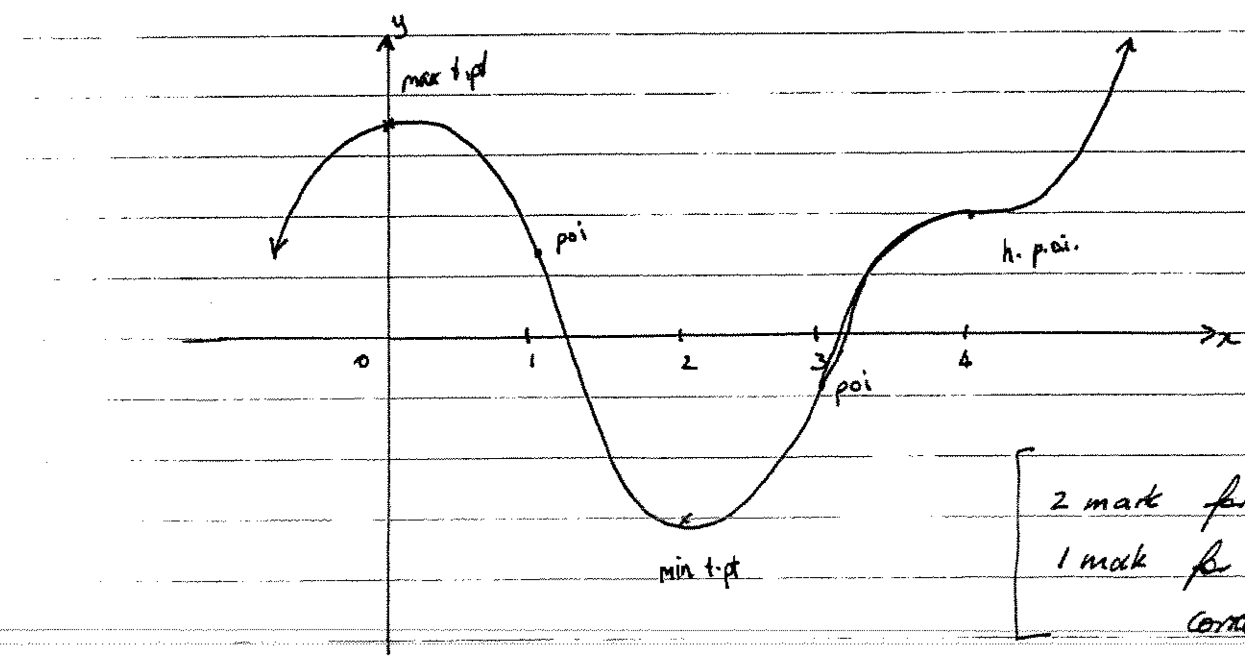
(b) St pts at $x = 0, 2$ or 4

Concave down when $x < 1$ ie. pt of inflexion $x = 1$

Concave up when $1 < x < 3$ ie. poi at $x = 3$

Concave down when $3 < x < 4$ ie. poi at $x = 4$.

Concave up when $x > 4$



[2 mark for correct graph
1 mark for 2/3 st. pts. correct.

(c) 3 Red, 4 Blue, 5 Yellow

(i) $P(2Y, R) = P(RYY) + P(YYR) + P(YRY)$

$= 3\left(\frac{3}{12} \times \frac{5}{11} \times \frac{4}{10}\right)$

$= \frac{3}{22}$

2mks (correct ans)
1mk (for correct arrangement)

(ii) 3 Red, 9 Non-red

$P(\text{at least one red}) = 1 - P(\text{no red})$

$= 1 - P(N, N, N)$

$= 1 - \left(\frac{9}{12} \cdot \frac{8}{11} \cdot \frac{7}{10}\right)$

$= 1 - \frac{21}{55}$

$= \frac{34}{55}$

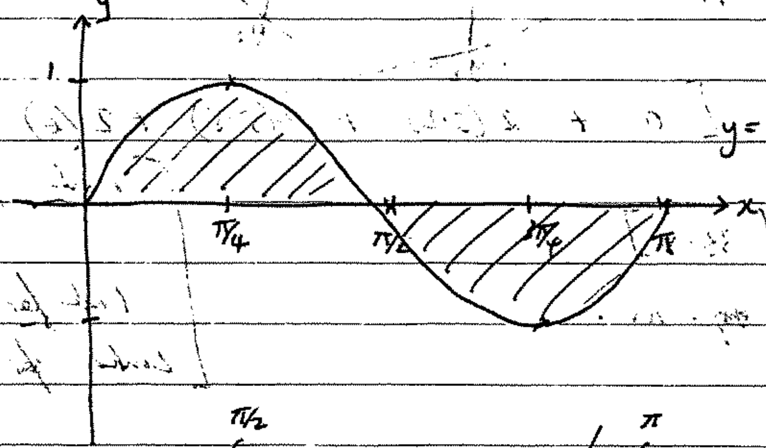
2 mk (correct ans)
1mk (correct arrangement)

Solutions Q7 Maths

(a) $y = \sin 2x$, Amplitude $= 1$

Period $= \frac{2\pi}{2} = \pi$.

(i)



[2 mks correct curve
1 mk Amplitude
1 mk Period]

(ii) Area $= \int_0^{\pi/2} \sin 2x \, dx + \left| \int_{\pi/2}^{\pi} \sin 2x \, dx \right|$

$= 2 \int_0^{\pi/2} \sin 2x \, dx$ (by symmetry).

$= 2 \left[-\frac{1}{2} \cos 2x \right]_0^{\pi/2}$

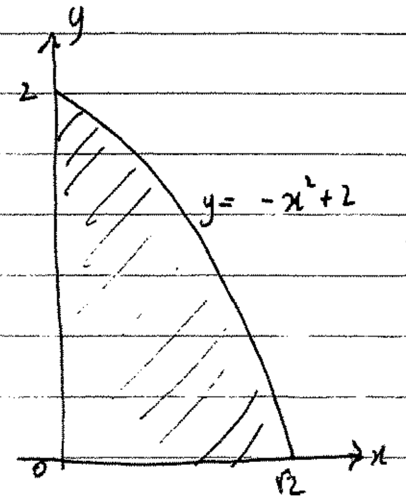
$= \left[-\cos \pi + \cos 0 \right]$

$= -(-1) + 1$

$= 2$ sq units..

[3 mks correct answer
1 mk for integral.
1 mk for integration
1 mk for substitution]

(b)



For $y = -x^2 + 2$

$\therefore x^2 = 2 - y$.

$\therefore$ Volume $= \pi \int_0^2 x^2 \, dy$.

$= \pi \int_0^2 (2 - y) \, dy$

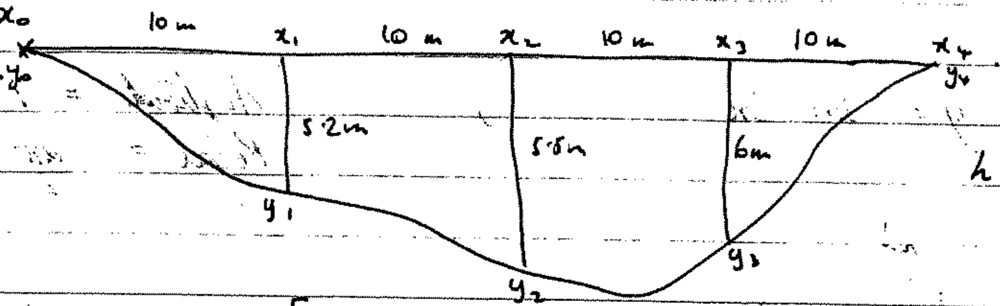
$= \pi \left[\left(2y - \frac{y^2}{2} \right) \right]_0^2$

$= \pi [2]$

$= 2\pi$ cubic units.

[3 mks for correct ans.
1 mk for correct integral
1 mk for correct integration
1 mk for correct subst.]

(c)



$h = 10$ m.

(i) Area $\doteq \frac{10}{2}\left[0 + 2(5.2) + 2(5.5) + 2(6) + 0\right]$.

$\doteq 5\,[33.4]$

$\doteq 167$ sq. m.

1 mk for formula.
1 mk for solution
2 mks for correct ans

(ii) The lack of accuracy of this method is the nature of the straight line does not match the curve. A better method is Simpson's Rule which uses a curve the approximate the gap.

or

More measurements by decreasing the value of h.

1 mk for straight line
1 mk for Simpson's Rule.
2 mks for correct ans.

Question ~

a) i) $5 = 15(1 - \sin 0.15t)$

$\frac{1}{3} = 1 - \sin 0.15t$ ✓

$-\frac{2}{3} = -\sin 0.15t$

$0.15t = \sin^{-1}(\frac{2}{3})$

$t = \frac{\sin^{-1}(\frac{2}{3})}{0.15}$

$= 4\text{hrs } 52\text{mins}$ ✓ (2)

4.865 (292mins)

ii) $\frac{dx}{dt} = 15 - 15\sin 0.15t$

$x = \int 15 - 15\sin 0.15t \, dt$ /

$= 15t + \frac{15}{0.15}\cos 0.15t + c$

$x=0 \quad t=0$

$0 = 0 + 100\cos 0 + c$

$c = -100$ /

$x = 15t + 100\cos 0.15t - 100$

$t = 3$

$x = 45 + 100\cos 0.45 - 100$

$= 35.045\text{km}$ ✓ (3)

b) $3^{2x} + 2 \times 3^x - 15 = 0$

Let $u = 3^x$ /

$u^2 + 2u - 15 = 0$

$(u+5)(u-3) = 0$

$u = -5$ or 3

$3^x = -5 \quad 3^x = 3$

No soln / $\quad x = 1$ ✓ (3)

c) $\Delta = b^2 - 4ac$

$= (k-2)^2 - 4 \times 1 \times 4$

$= k^2 - 4k + 4 - 16$ ✓

$= k^2 - 4k - 12$

i) $\Delta = 0$

$k^2 - 4k - 12 = 0$

$(k-6)(k+2) = 0$ ✓ (2)

$k = 6$ or -2

ii) $\Delta > 0$

$(k-6)(k+2) > 0$ ✓

-2

$k > 6$ or $k < -2$ ✓ (2)

Q9 (a) $\frac{1}{2}\ln(12-x) = \ln x$

$\therefore \ln(12-x) = 2\ln x$

$\therefore \ln(12-x) = \ln x^2$

$\therefore 12-x = x^2$

$\therefore x^2+x-12=0$

$\therefore (x+4)(x-3)=0$

$\therefore x=-4$ or 3 — 1 Mark - solved quadratic

but $\ln -4$ is undefined

$\therefore x = 3$ — 1 Mark - discarded -4

(b)(i) $r = \frac{0.06}{12} = 0.005$ per month

$\therefore A_1 = P(1.005) - 3750$

$\therefore A_2 = [P(1.005)-3750](1.005) - 3750$

$= P(1.005)^2 - 3750(1.005) - 3750$ — 1 Mark

(ii) $\therefore A_n = P(1.005)^n - 3750(1.005)^{n-1} - \ldots - 3750(1.005) - 3750$

$= P(1.005)^n - 3750[1.005^{n-1} + \ldots + 1.005 + 1]$ — 1 Mark

$= P(1.005)^n - 3750\left(\frac{1.005^n - 1}{1.005 - 1}\right)$

$= P(1.005)^n - \frac{3750}{0.005}(1.005^n - 1)$

$= P(1.005)^n - 750000(1.005^n - 1)$ — 1 Mark

(iii) $A_{240} = 0$ — 1 Mark

$\therefore 0 = P(1.005)^{240} - 750000(1.005^{240} - 1)$

$\therefore P = \frac{750000(1.005^{240}-1)}{1.005^{240}}$

$= \$523427.89$ — 1 Mark

Q9 (c)(i) $A = A_0 e^{-kt}$

$\therefore \dfrac{dA}{dt} = -k A_0 e^{-kt}$

$= -kA$ 1 Mark

(ii) $T = 500$ $A = \frac{A_0}{2}$

$\therefore \dfrac{A_0}{2} = A_0 e^{-500k}$ 1 Mark

$\therefore e^{-500k} = \frac{1}{2}$

$\therefore -500k = \ln \frac{1}{2}$

$$\left.\begin{aligned} \therefore k &= \frac{-\ln \frac{1}{2}}{500} \\ &= \frac{\ln 2}{500} \\ &= 0.001386 \end{aligned}\right\} \text{all accepted}$$ 1 Mark

(iii) $A = 0.15 A_0$

$\therefore 0.15 A_0 = A_0 e^{-kt}$ 1 Mark

$\therefore 0.15 = e^{-kt}$

$\therefore -kt = \ln 0.15$

$\therefore t = -\dfrac{\ln 0.15}{k}$

$= 1368.48$

≈ 1400 years 1 Mark

Note: accepted answers based on mistakes in part (ii)

Question 10

a) AC $\quad M_{AC} = \dfrac{a-c}{4-1} = \dfrac{a-c}{5}$

i) $y - y_1 = m(x - x_1) \quad (4, a)$

$y - a = \frac{a-c}{5}(x-4)$

$y = \frac{a-c}{5}(x-4) + a$

ii) Midpoint $\left(\frac{1+5}{2}, \frac{b+d}{2}\right)$

$= \left(3, \frac{b+d}{2}\right)$

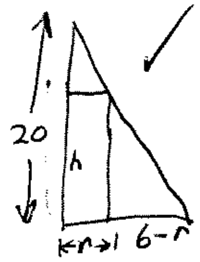
iii) $\dfrac{b+d}{2} = \dfrac{a-c}{5}(3-4) + a$

$b + d = \dfrac{-2(a-c)}{5} + 2a$

$b + d = \dfrac{-2a + 2c + 10a}{5}$

$b = \dfrac{8a+2c}{5} - d = \dfrac{2c + 8a - 5d}{5}$

b) i)



$\dfrac{6-r}{6} = \dfrac{h}{20}$

$h = \dfrac{20(6-r)}{6}$

$V = \pi r^2 h$

$= \pi r^2 \times \dfrac{20(6-r)}{6}$

$= \dfrac{10\pi r^2(6-r)}{3}$

ii) $V = \dfrac{60\pi r^2 - 10\pi r^3}{3}$

$\dfrac{dV}{dr} = \dfrac{120\pi r}{3} - \dfrac{30\pi r^2}{3}$

$= 40\pi r - 10\pi r^2$

$0 = 10\pi r(4 - r)$

$r = 0$ or 4

$\dfrac{d^2V}{dr^2} = 40\pi - 20\pi r$

when $r = 4$ $\dfrac{d^2V}{dr^2} = -40\pi < 0$

$\therefore$ max

$\therefore r = 4$

$h = \dfrac{20(6-4)}{6}$

$= \dfrac{20 \times 2}{6} = \dfrac{40}{6}$

iii) $\therefore V = \pi(4)^2\left(\frac{40}{6}\right)$

$= 106\frac{2}{3}\pi$ units.

≈ 335.1